Recommendations and Conclusions

The use of method 2a is recommended for two-dimensional look-up situations. It is the fastest method and carries little additional storage requirements for the polynomial coefficients. In three-dimensional look-up cases, the use of method 2b is recommended, but only if the independent variables are moving into new subregions on 1 or 2% of the total number of calls. If the independent variable subregions are changing more rapidly, then the use of method 2a is suggested. This increases the storage requirements and represents a tradeoff of memory for speed. Neither of methods 2 are recommended for a four-dimensional (or higher) look-up routine. Method 2a drastically increases storage requirements, and method 2b results in an unacceptably high frame time when a new independent variable subregion is entered.

The development of these new algorithms supplies the simulation engineer with another set of tools for table look-up applications. Like any tools, they must be used carefully and only in the appropriate situations. All of the methods described here produce the same numerical result, but the new algorithms presented require fewer floating point calculations during certain real-time portions of the simulation. When used in appropriate situations, these algorithms can significantly reduce look-up frame times.

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Connection Between Leading-Edge Sweep, Vortex Lift, and Vortex Strength for Delta Wings

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Nomenclature

A = coefficient of power-law function
C_L = total lift coefficient referenced to planform area
$C_{L,nl}$ = nonlinear component of lift coefficient, $C_L - K_p\alpha$
$C_{L,p}$ = component of lift coefficient due to attached flow without leading-edge suction
$C_{L,v}$ = vortex lift component
f,g = similarity functions
K = Sychev similarity parameter, $\tan\alpha/\tan\epsilon$
K_p = slope of lift coefficient at zero angle of attack
K_v = suction analogy coefficient for separated flow
ℓ = length of wing centerline chord
ℓ_{le} = length of delta wing leading edge
M_∞ = Mach number
S_p = wing planform area
s = semispan of wing
α = angle of attack
Γ = vortex circulation
ϵ = wing apex half angle
Λ = leading-edge sweep

Introduction

THE purpose of this Note is to clarify the effect of leading-edge sweep on the vortex lift and leading-edge vortex strength of a slender wing. It is often believed that an increase in sweep increases both the vortex lift and the vortex strength. However, the opposite is true. Using the suction analogy,[1] similarity,[2] and numerical and experimental data, we derive simple formulas giving the actual dependence for delta wings. Some important aspects of leading-edge vortex lift as delineated by Polhamus[1] are reviewed first.

Difference Between Vortex and Nonlinear Lift

As part of his work on the leading-edge suction analogy for sharp-edged delta wings, Polhamus analyzed the effect of the loss of leading-edge suction due to leading-edge separation.[1] He showed that the variation with angle of attack of the zero-suction attached-flow lift is given by

$$C_{L,p} = K_p \sin\alpha \cos^2\alpha \tag{1}$$

where K_p is used to designate the lift-coefficient slope at zero angle of attack. Hence, as defined by Polhamus, the lift that should be associated with the leading-edge vortex (i.e., the so-called vortex lift $C_{L,v}$) is given by

$$C_{L,v} = C_L - K_p \sin\alpha \cos^2\alpha \tag{2}$$

Unfortunately, $C_{L,v}$ is often confused with the so-called nonlinear increment of lift $C_{L,nl}$, which is given by

$$C_{L,nl} = C_L - K_p\alpha \tag{3}$$

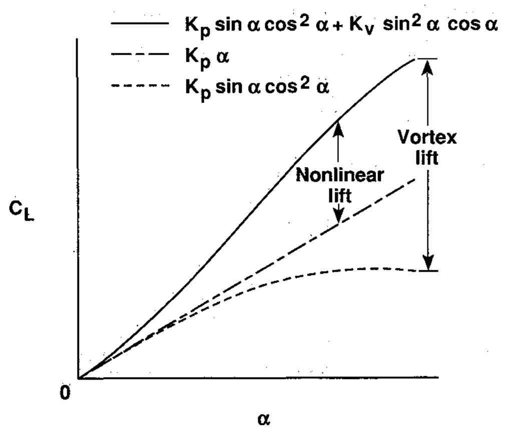

Fig. 1 Difference between vortex lift and nonlinear lift.

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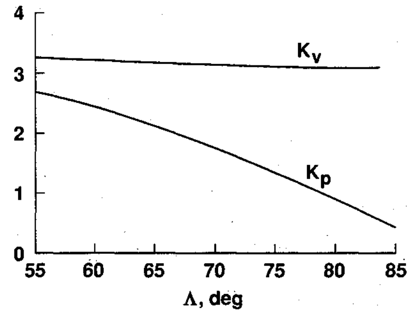

Fig. 2 Linear-theory values of K_p and K_v for delta wings at low speeds.[1]

The difference between the two lift increments is illustrated in Fig. 1.

The different behaviors for $C_{L,nl}$ and $C_{L,v}$ can be illustrated conveniently using the suction analogy,[1] which is known to be accurate for slender delta wings at angles of attack below the onset of vortex breakdown. The equation for lift is

$$C_L = K_p \sin\alpha \cos^2\alpha + K_v \sin^2\alpha \cos\alpha \tag{4}$$

where K_v is related directly to the coefficient for leading-edge suction. Polhamus[1] showed that the second term of Eq. (4) represents the lift that should be attributed to the leading-edge vortices $C_{L,v}$. The nonlinear lift according to the analogy is obtained by substituting Eq. (4) into Eq. (3) to get

$$C_{L,nl} = K_p (\sin\alpha \cos^2\alpha - \alpha) + K_v \sin^2\alpha \cos\alpha \tag{5}$$

Differentiating Eq. (5) with respect to Λ gives

$$\frac{\partial C_{L,nl}}{\partial \Lambda} = (\sin\alpha \cos^2\alpha - \alpha) \frac{\partial K_p}{\partial \Lambda} + \sin^2\alpha \cos\alpha \frac{\partial K_v}{\partial \Lambda} \tag{6}$$

Linear-theory values of K_p and K_v for delta wings are given by Polhamus,[1] as shown in Fig. 2, and it is easily determined that the right-hand side of Eq. (6) is positive for the angle-of-attack range for which significant vortex lift is generated. Hence, the nonlinear lift for delta wings increases with increasing sweep.

Differentiating the vortex lift gives

$$\frac{\partial C_{L,v}}{\partial \Lambda} = \sin^2\alpha \cos\alpha \frac{\partial K_v}{\partial \Lambda} \tag{7}$$

Note that K_v decreases asymptotically, albeit very slowly, to π with increasing sweep. Hence, the vortex lift for delta wings decreases with increasing sweep. The behavior of $C_{L,nl}$ and $C_{L,v}$ is illustrated in Fig. 3 for delta wings at $\alpha = 20$ deg.

Vortex Strength

The authors have been unable to find a sufficiently large systematic data base with which to examine directly the dependence of leading-edge vortex strength on leading-edge sweep. However, similarity together with data on vortex strength variation with angle of attack can be used. Hemsch[2] has demonstrated that the high angle-of-attack similarity of Sychev[3] correlates the vortex properties of sufficiently slender wings. Furthermore, he showed that the Sychev similarity gives the following result for the leading-edge-vortex strength at the trailing edge of a slender wing

$$\frac{\Gamma}{sU_\infty \sin\alpha} = f(K, M_\infty) \tag{8}$$

where $K = \ell \tan\alpha / s$. The function f is the same for all wings in a given affine family and must be found from experiment or computations. With a specialization to delta wings so that $\tan\epsilon = s/\ell$, Eq. (8) can be rewritten as

$$\frac{\Gamma / U_\infty \ell}{K \tan^2\epsilon \cos\alpha} = f(K, M_\infty)$$

or

$$\frac{\Gamma}{U_\infty \ell} = \tan^2\epsilon \cos\alpha \, g(K, M_\infty) \tag{9}$$

where $g = Kf$ and is, therefore, also a similarity function.

The function g was estimated from the numerical conical slender-body-theory results of Smith[4] that are shown logarithmically in Fig. 4. (Note that Smith used the small-angle approximations of the trigonometric functions in the similarity relations.) A power-law curve of the form

$$g = AK^{1.2} \tag{10}$$

where A is a constant, accurately fits the Smith numerical results as shown by the solid line in Fig. 4.

The function g was computed from the low-speed data of Wentz and McMahon[5] for a 62-deg sweep delta wing at several angles of attack. The result is given in Fig. 4. Shown also are estimates derived from the low-speed data of Delery et al.[6] for a 75-deg sweep delta wing at various angles of attack. The authors of Ref. 6 did not compute the vortex circulation, and it was necessary to estimate it by assuming the vortex to be axisymmetric downstream of the trailing edge and using the velocity profiles taken through the core. Equation (10) fits the

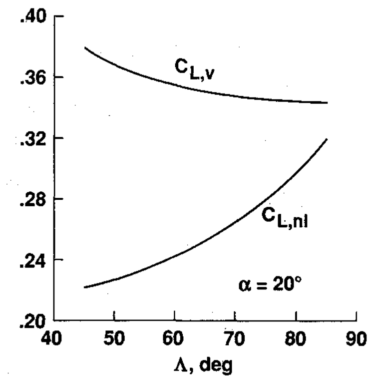

Fig. 3 Behavior of nonlinear and vortex lift coefficients for delta wings.

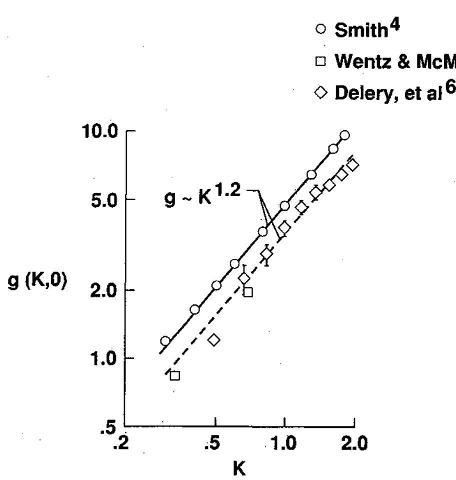

Fig. 4 Estimates of the similarity function g for delta wings.

experimental data satisfactorily, with a different coefficient, as shown by the dashed line.

Given the above results, it is reasonable to use Eq. (10) to estimate the effects of sweep. Substituting Eq. (10) into Eq. (9) gives

$$\frac{\Gamma}{U_\infty} \sim \ell\,(\tan\epsilon)^{0.8} = \frac{\ell}{(\tan\Lambda)^{0.8}} \tag{11}$$

for fixed α, demonstrating that the leading-edge vortex strength of delta wings with the same length decreases with increasing leading-edge sweep. Equation (11) can be expressed in terms of the planform area as

$$\frac{\Gamma}{U_\infty} \sim \frac{\sqrt{S_p}}{(\tan\Lambda)^{0.3}} \tag{12}$$

which shows that the previous statement is also true if S_p is held constant. We also can write

$$\frac{\Gamma}{U_\infty} \sim \ell_{le}\frac{(\tan\Lambda)^{0.2}}{\sqrt{1+\tan^2\Lambda}} \tag{13}$$

where ℓ_{le} is the length of the wing leading edge. Using Eq. (13), it is easily shown that Γ also decreases with increasing Λ if ℓ_{le} is held constant ($\Lambda > 26.6$ deg). This is a useful result for variable sweep wings.

Concluding Remarks

In this Note the difference between nonlinear lift and vortex lift has been distinguished. The nonlinear lift is the difference between the actual lift at a given α and that given by $K_p\alpha$. The vortex lift is the increment of lift above the zero leading-edge suction attached-flow lift and is due to the presence of the leading-edge vortex. Through the use of similarity and several experimental and computational results, it has been possible to show that the effect of increasing leading-edge sweep for slender delta wings is to decrease vortex lift and leading-edge vortex strength. These results are useful for the development of future design strategies for high angle-of-attack wing performance when it is desired to tailor the growth of the leading-edge vortices using sweep, camber, and leading-edge shape.

Acknowledgments

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Nonlinear Effects in the Two-Dimensional Adaptive-Wall Outer-Flow Problem

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Introduction

THE purpose of this Note is to examine the importance of nonlinear effects on the solution to the two-dimensional adaptive-wall outer-flow problem. The Note compares outer-flow solutions computed using the transonic small perturbation (TSP) equation with solutions based on the linear Prandtl-Glauert equation. Both methods are applied to simulated measurements of transonic flow past a two-dimensional airfoil in free air.

Wall settings for free-air flow are established in an adaptive-wall wind tunnel using flow measurements at or near the walls of the test section without any information about the model. This is possible since, in free air, redundant velocity distributions—for example, orthogonal components of velocity along a contour surrounding the model—are uniquely related. Thus, the walls can be adjusted until measured flow conditions satisfy these free-air relationships.[1]

The free-air relationships are derived by solving an outer-flow problem that is, in effect, a mathematical extension of the wind-tunnel flow from the contour to infinity. For a two-dimensional, rectangular contour extending upstream and downstream to infinity, the problem becomes that of solving for the flow in separate, infinite half-planes, one above and the other below the model, each subject to measured boundary conditions along the edge of the plane and free-air boundary conditions at infinity (vanishing perturbations).

Linear theory can be used to accurately represent most outer flows up to low transonic speeds, including many cases where flow near the model is quite nonlinear.[2,3] This is possible since perturbations, and thus nonlinear effects, are much smaller in the outer region than they are near the model. Linear solutions are convenient since they implicitly satisfy the free-air condition at infinity, can be derived analytically and expressed in closed form, and can be evaluated very quickly by a small computer.

It is inevitable that beyond some freestream Mach numbers, linear outer solutions will be inadequate. Unfortunately, there are no analytic solutions to even the simplest nonlinear equation, so approximate solutions can only be estimated, numerically or by other means. Since numerical solutions are computed in a finite domain, the free-air boundary condition at infinity cannot be applied directly. One approach to this problem is to transform the infinite physical domain into a finite computational domain.[4] Alternatively, if the numerical domain is simply a subset of the physical domain, either it must be large enough that use of zero perturbations provides a good approximation along its far-field boundaries or nonzero conditions must be estimated along less remote boundaries. Two two-dimensional experiments that used the large-domain approach have been reported.[4–6]

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